Calculating Lattice Energies Using the Born-Haber Cycle
An Extension Activity for AP Chemistry Students

A particular set of equations known as a Born-Haber cycle demonstrates how chemists are able to use the first law of thermodynamics—that the energy of the universe is conserved in any chemical or physical change—to find an unknown energy value that is difficult or impossible to measure experimentally. Some energy quantities, such as the lattice energy of a mineral or the electron affinity of an atom, can be difficult to measure in the lab. Examining a Born-Haber cycle we see that there is more than one path to the formation of a substance in a particular state, and that if we use consistent definitions, an energy value that we seek can be calculated from energy values that we already know.

The following exercise will help us see the way these energy values relate to one another, give us practice with their definitions and symbols, and deepen our insight to their meaning when we see them in other types of problems.

Each physical or chemical change represented has:

- an equation that represents a clearly defined physical or chemical change;
- a definition of the particular type of energy change;
- a symbol or abbreviation for the energy change equal to a value for the change in enthalpy (ΔH), the energy that is released or absorbed during the change, expressed in kJ/mol; and
- a name by which that change in enthalpy is commonly known.

To set up the equations in a Born-Haber cycle, cut out the cards for names, equations, definitions, and symbols with energy values. Arrange them to show the two alternate pathways to forming the ionic solid, linking the sequence of changes using the equation cards, and placing the definitions, names, and values near each equation. Pay close attention to the physical phases noted in the definitions.

When the cards have been arranged, examine the way the equations fit together. Can you clearly trace two paths to a final product? If so, according to the first law of thermodynamics, the energy changes along one path will be equal to the energy changes along the other path. By setting the sum of energy changes from one path equal to the energy changes of the other path, find the unknown value for the lattice energy of the solid.
CYCLE 1

\[
\text{Li (s) } + \frac{1}{2} \text{F}_2 (g) \rightarrow \text{LiF(s)}
\]

\[
\text{Li}^+ (g) + \text{F}^- (g) \rightarrow \text{LiF(s)}
\]

\[
\frac{1}{2} \text{F}_2 (g) \rightarrow \text{F}^- (g)
\]

\[
\text{Li (g)} \rightarrow \text{Li}^+ (g) + \text{e}^-
\]

\[
\text{Li (s)} \rightarrow \text{Li(g)}
\]

\[
\text{F (g)} + \text{e}^- \rightarrow \text{F}^- (g)
\]
\[ \Delta H^\circ_{\text{LiF}} = -594.1 \text{ kJ} \]

\[ \Delta H_{\text{sub}_{\text{Li}}} = 155.2 \text{ kJ} \]

1st IE = 520 kJ

\[ \frac{1}{2} \text{BE}_{\text{F}_2} = 75.3 \text{ kJ} \]

\[ \text{EA}_\text{F} = -328 \text{ kJ} \]

\[ \text{LE}_{\text{LiF}} = ? \]
The change in enthalpy when one mole of a substance in its standard state is formed from its constituent elements in their standard states.

The energy required to remove the outermost electron of each atom in one mole of an element in its gaseous state.

The energy released or absorbed when an electron is added to each atom in one mole of a substance in its gaseous state.

The energy needed to transform one mole of a substance from the solid to the gaseous state.

The energy released when one mole of an ionic compound is formed from its constituent ions in their gaseous states.

The energy required to break one mole of bonds between two atoms.
Standard enthalpy of formation

First ionization energy

Lattice energy

Bond energy

Enthalpy of sublimation

Electron affinity
### CYCLE 2

\[
\text{Na (s)} + \frac{1}{2} \text{Cl}_2 (g) \rightarrow \text{NaCl (s)}
\]

\[
\text{Na}^+ (g) + \text{Cl}^- (g) \rightarrow \text{NaCl (s)}
\]

\[
\frac{1}{2} \text{Cl}_2 (g) \rightarrow \text{Cl} (g)
\]

\[
\text{Na} (g) \rightarrow \text{Na}^+ (g) + e^-
\]

\[
\text{Na (s)} \rightarrow \text{Na} (g)
\]

\[
\text{Cl} (g) + e^- \rightarrow \text{Cl}^- (g)
\]
\[ \Delta H^\circ_{\text{NaCl}} = -411 \text{ kJ} \]

\[ \Delta H_{\text{sub}_{\text{Na}}} = 108 \text{ kJ} \]

\[ 1\text{st IE}_{\text{Na}} = 496 \text{ kJ} \]

\[ \frac{1}{2} \text{BE}_{\text{Cl}_2} = 75.3 \text{ kJ} \]

\[ \text{EA}_{\text{Cl}} = -349 \text{ kJ} \]

\[ \text{LE}_{\text{NaCl}} = ? \]
CYCLE 3

For cycle 3, set up the Born-Haber cycle to find lattice energy using known energy values as before. This time you will also need to find the second ionization energy of calcium where:

Second ionization energy

The energy required to remove the second outermost electron of each atom in one mole of an element in its gaseous state.

\[ \text{Ca}^+ (g) \rightarrow \text{Ca}^{2+} (g) + e^- \]

\[ 2\text{nd IE}_{\text{Ca}} = 1145 \text{ kJ} \]
\[ \text{Ca (s)} + \text{Cl}_2 (g) \rightarrow \text{CaCl}_2 (s) \]

\[ \text{Ca}^{2+} (g) + 2\text{Cl}^- (g) \rightarrow \text{CaCl}_2 (s) \]

\[ \text{Cl}_2 (g) \rightarrow 2\text{Cl} (g) \]

\[ \text{Ca (g)} \rightarrow \text{Ca}^+ (g) + \text{e}^- \]

\[ \text{Ca (s)} \rightarrow \text{Ca (g)} \]

\[ 2\text{Cl} (g) + 2\text{e}^- \rightarrow 2\text{Cl}^- (g) \]
$$\Delta H^\circ_{f_{CaCl_2}} = -795.8 \text{ kJ}$$

$$\Delta H_{\text{sub}_{Ca}} = 178.2 \text{ kJ}$$

$$1\text{st IE}_{Ca} = 590 \text{ kJ}$$

$$\frac{1}{2} \text{BE}_{Cl_2} = 244 \text{ kJ}$$

$$\text{EA}_{Cl} = -349 \text{ kJ}$$

$$\text{LE}_{CaCl_2} = ?$$
**TEACHER’S KEY**

- In this exercise, students will identify ionization energy, electron affinity, standard enthalpy of formation, energy of sublimation, bond energy, and lattice energy as $\Delta H$ values for particular reactions.
- Constructing a Born-Haber cycle will demonstrate how Hess’s law (and the first law of thermodynamics) can be used to find one of the energy values in the cycle, if the others are known.
- Creating a visual arrangement with cards gives students a visual context to help them see how the specific energy quantities can be used to determine the unknown value.
- As shown below, definitions, symbols, equations, and energy values could each be copied in a different color, to help with recognizing and organizing the cards.
- Students should work with one complete set of equations at a time.

Sample of student arrangement:
### Cycle 1

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Standard Enthalpy of Formation of LiF (s)</th>
<th>( \Delta H_{\text{sub}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Li (s) → Li (g)</td>
<td>111</td>
<td>155.2 kJ</td>
</tr>
<tr>
<td>Li (g) → Li⁺(g) + e⁻</td>
<td>1st Ionization Energy of Li = 520 kJ</td>
<td></td>
</tr>
<tr>
<td>½ F₂ (g) → F (g)</td>
<td>½ Bond Energy of F₂ = 75.3 kJ</td>
<td></td>
</tr>
<tr>
<td>F (g) + e⁻ → F⁻ (g)</td>
<td>Electron Affinity for fluorine = −328 kJ</td>
<td></td>
</tr>
<tr>
<td>Li⁺ (g) + F⁻ (g) → LiF (s)</td>
<td>Lattice Energy of LiF = ??</td>
<td></td>
</tr>
</tbody>
</table>

\(-594 \text{ kJ} = LE + −328 \text{ kJ} + 75.3 \text{ kJ} + 520 \text{ kJ} + 155.2 \text{ kJ} \)

*Lattice Energy for LiF = −1016 kJ*

### Cycle 2

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Standard Enthalpy of Formation of NaCl (s)</th>
<th>( \Delta H_{\text{sub}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Na (s) → Na (g)</td>
<td>111</td>
<td>108 kJ</td>
</tr>
<tr>
<td>Na (g) → Na⁺(g) + e⁻</td>
<td>1st Ionization Energy of Na = 496 kJ</td>
<td></td>
</tr>
<tr>
<td>½ Cl₂ (g) → Cl (g)</td>
<td>½ Bond Energy of Cl₂ = 122 kJ</td>
<td></td>
</tr>
<tr>
<td>Cl (g) + e⁻ → Cl⁻ (g)</td>
<td>Electron Affinity for chlorine = −349 kJ</td>
<td></td>
</tr>
<tr>
<td>Na⁺ (g) + Cl⁻ (g) → NaCl (s)</td>
<td>Lattice Energy of NaCl = ??</td>
<td></td>
</tr>
</tbody>
</table>

\(-411 \text{ kJ} = LE + −349 \text{ kJ} + 122 \text{ kJ} + 496 \text{ kJ} + 108 \text{ kJ} \)

*Lattice Energy for NaCl = −788 kJ*

### Cycle 3

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Standard Enthalpy of Formation of CaCl₂ (s)</th>
<th>( \Delta H_{\text{sub}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ca (s) → Ca (g)</td>
<td>111</td>
<td>178.2 kJ</td>
</tr>
<tr>
<td>Ca (g) → Ca⁺(g) + e⁻</td>
<td>1st Ionization Energy of Ca = 590 kJ</td>
<td></td>
</tr>
<tr>
<td>Ca⁺ (g) → Ca²⁺(g) + e⁻</td>
<td>2nd Ionization Energy of Ca = 1145 kJ</td>
<td></td>
</tr>
<tr>
<td>Cl₂ (g) → 2Cl (g)</td>
<td>Bond Energy of Cl₂ = 244 kJ</td>
<td></td>
</tr>
<tr>
<td>2Cl (g) + 2e⁻ → 2Cl⁻</td>
<td>Electron Affinity for chlorine = 2(−349 kJ)</td>
<td></td>
</tr>
<tr>
<td>Ca²⁺ (g) + 2Cl⁻ (g) → CaCl₂ (s)</td>
<td>Lattice Energy of CaCl₂ = ??</td>
<td></td>
</tr>
</tbody>
</table>

\(-795.8 \text{ kJ} = LE + 2(−349 \text{ kJ}) + 244 \text{ kJ} + 1145 \text{ kJ} + 590 \text{ kJ} + 178.2 \text{ kJ} \)

*Lattice Energy for CaCl₂ (s) = −2255 kJ*
Further Analysis

Lattice energy is often used to estimate the strength of an ionic bond. Comparing the lattice energies of the three salts, students could now be asked to look at the relative strengths of the bonds in the compounds to relative sizes of ions and relative charge on ions.

According to Coulomb’s law, the strength of the bond should increase with increasing charge on the ion, and with decreasing size of the ion.

Do students see any evidence for this in their results? Can they explain their reasoning?